



ORIGINAL RESEARCH ARTICLE

SHEAR RELIABILITY OF REINFORCED CONCRETE BEAMS: A FORM-MONTE CARLO FRAMEWORK

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ARTICLE INFORMATION

Received: 29th April 2026
Revised: 10th May 2026
Accepted: 10th May 2026

Keywords:

FORM

Limit state function

Reliability index

Structural reliability

Shear failure

ABSTRACT

Shear failure in reinforced concrete (RC) beams is a brittle and often sudden event that poses serious risks to structural safety. While ACI 318-19 and Eurocode 2 provide deterministic shear design provisions, their embedded partial safety factors do not explicitly propagate the statistical variability of resistance and demand variables, resulting in a design space where reliability levels vary substantially from one beam configuration to another. This study develops a fully integrated probabilistic framework that combines the First-Order Reliability Method (FORM) with large-scale Monte Carlo Simulation (MCS, $N = 100,000$) to quantify shear failure reliability across a parametric space spanning concrete compressive strengths of 20 MPa to 60 MPa, stirrup reinforcement ratios of 0 to 0.004, and shear span-to-depth ratios of 1.5 to 5.0. All basic variables, covering material, geometric, load, and model uncertainty were assigned probability distributions calibrated from experimental databases and the JCSS Probabilistic Model Code. Additionally, Model uncertainty was characterised from 47 beam tests selected from the ACI-DAfStb database. The results show that, for the Compression Chord Capacity Model (CCCM), the calibrated parameters are bias = 1.01 and COV = 0.13. Similarly, the ACI 318-19 model, which is used as the primary design basis in this study, the validation yields bias = 1.28 and COV = 0.29, reflecting its highest scatter. However, the main results used CCCM-calibrated model uncertainty as the more accurate bound. It was observed that the computed reliability indices ranged from 2.04 to 5.12, with beams lacking transverse reinforcement falling below the code target of 3.5 at all shear span-to-depth ratios. In addition, Live load variability was the dominant uncertainty source (32 to 35% of total variance), followed by concrete strength variability (21 to 24%) and model uncertainty (15 to 16%) respectively. A sensitivity analysis confirmed that the results are robust to plausible distribution alternatives, while an elevated concrete coefficient of variation of 0.20, a representative of Nigerian construction site conditions, reduced the reliability index by approximately 0.30 units. Therefore, a reliability-consistent shear partial safety factor of approximately 1.58 is recommended, compared to the ACI equivalent of 1.33. Interestingly, it is clear that the study showed direct implications for code calibration under Nigerian Standards (NIS 369), and the open-source simulation framework is available to support national code revision efforts.

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1.0 Introduction

Shear failure in reinforced concrete beams is a brittle and often sudden event that offers little visible warning compared to the gradual deflection associated with flexural distress. The consequences of shear collapse can be severe in bridge and building failures largely attributed to deficient transverse reinforcement or inadequate shear capacity in slender members (Casas and Gomez, 2013; Val and Stewart, 2020). The complex nature of

shear transfer, involving aggregate interlock along diagonal cracks, dowel action of longitudinal reinforcement, arch action in deep beams, and the truss mechanism of stirrups, has historically defied simple unified treatment, producing significant scatter among competing predictive models (Cladera *et al.*, 2016; Collins *et al.*, 2020). The truss analogy for shear, originally formulated by Ritter (1899) and Morsch (1909), and later extended to strut-and-tie methods by Schlaich *et al.* (1987), remains the conceptual foundation of modern code provisions. The Modified Compression Field Theory (MCFT) developed by Vecchio and Collins (1986) and further advanced by Collins and Mitchell (1980) provides a more rigorous field-equilibrium basis for shear prediction.

Major design codes, including ACI 318-19 (ACI Committee 318, 2019), EN 1992-1-1 (CEN, 2004), and CSA A23.3-19 (CSA, 2019), address shear design through semi-empirical expressions calibrated on experimental databases. These expressions are applied with partial safety factors intended to maintain a minimum safety margin. Because these factors were established through historical experience rather than explicit probabilistic analysis, the reliability levels produced are non-uniform across the beam design space. For instance, it has been shown that slender beams without transverse reinforcement may exhibit implied reliability indices well below the nominal code target of 3.5 (Pacheco *et al.*, 2021; Kala, 2021; Navrátil *et al.*, 2022).

In the Nigerian construction environment, however, concrete production quality may be subjected to greater variability than assumed in the ACI or Eurocode probabilistic bases. Also, site supervision practices and material supply chains could introduce additional uncertainty capable of widening the gap between nominal and achieved safety levels. As such, the Nigerian Industrial Standards (NIS 369:2004) draw heavily on British Standards and on Eurocode-compatible reliability assumptions, yet no systematic probabilistic study has been conducted to verify whether these assumed reliability levels are achieved under typical Nigerian material and loading conditions. In their study, Adewuyi and Adegoke (2008) reported the variability of an in-situ concrete strength in Nigerian practice, underscoring the relevance of this gap.

Reliability-based methods, including FORM and MCS, provide the mathematical framework needed to propagate uncertainty through the design process and compute explicit failure probabilities. Similarly, Melchers and Beck (2018) provide a comprehensive treatment of these methods. Furthermore, Wang *et al.* (2018) compared Monte Carlo and importance sampling approaches, while Feng *et al.* (2023) demonstrated the utility of surrogate-model-assisted reliability analysis. Their systematic application to the shear limit state in RC beams remains limited in scope compared to flexural and seismic reliability (Kappos and Sextos, 2015). Recently, Lima *et al.* (2022) worked on model uncertainty effects while Vrdoljak *et al.* (2021) reported the results on the statistical characterisation of shear resistance. In addition, Yang and Ma (2021) further dwelled on the multi-mode reliability. For deep beams, Park *et al.* (2019) and Nguyen *et al.* (2020) provide probabilistic assessments using strut-and-tie and response surface methods respectively. It is noteworthy that the influence of model uncertainty on partial factor calibration has been addressed for nonlinear analysis applications by Gino *et al.* (2021) and Castaldo *et al.* (2022).

Therefore, this study addresses the gap established here by developing and applying a fully integrated probabilistic framework that: (i) models all principal uncertainty sources in shear design, covering material, geometric, load, and model uncertainty; (ii) implements and cross-validates both FORM and MCS; (iii) generates reliability index surfaces over the full parametric design space; (iv) quantifies the sensitivity of results to distribution assumptions and inter-variable correlations; and (v) discusses implications specifically for the Nigerian construction environment and for code calibration bodies.

2. Materials and Method

This section presents the materials, theoretical formulations, reliability models, and computational procedures that will be employed to investigate the shear reliability of reinforced concrete (RC) beams using a coupled First-Order Reliability Method (FORM) and Monte Carlo Simulation (MCS) framework. The methodology is developed to quantify the probability of shear failure and evaluate the reliability performance of RC beams subjected to uncertainties in material properties, geometric parameters, reinforcement characteristics, and applied loads.

2.1 Overall Framework

Figure 1 presents the stepwise simulation framework adopted in this study. The process begins with problem definition, and then statistical characterisation of all basic random variables, limit state formulation, FORM and MCS computation, validation against experimental data, parametric sweeping, and finally interpretation relative to code safety targets.

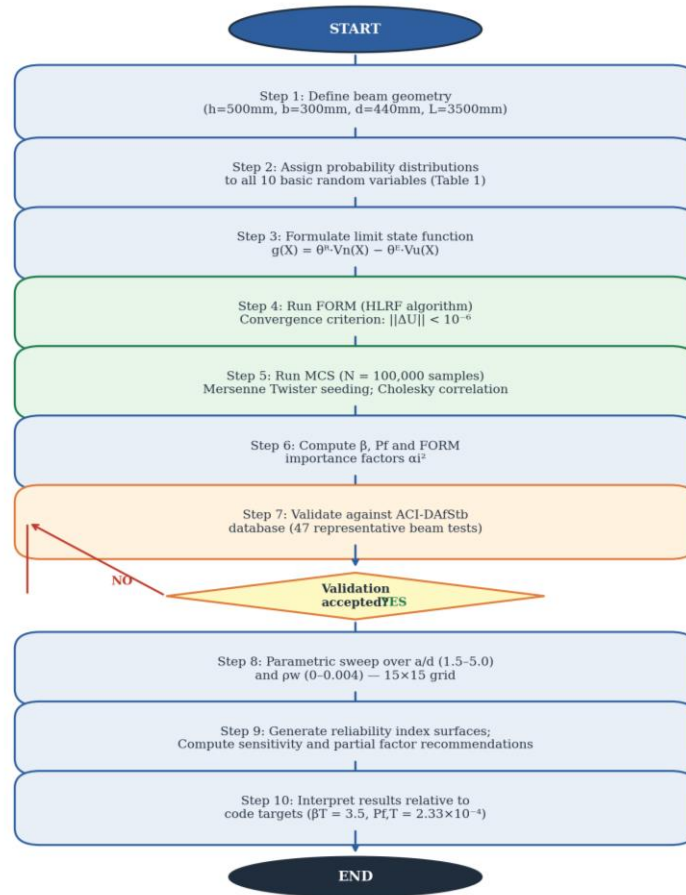


Figure 1: Flowchart of the probabilistic shear reliability assessment framework

2.2 Beam Configuration

A simply supported rectangular RC beam under a central point load is considered. This configuration is the most extensively represented in experimental shear databases. The nominal geometry is: total depth $h = 500$ mm, width $b = 300$ mm, effective depth $d = 440$ mm, and span $L = 3,500$ mm, giving a baseline shear span-to-depth ratio of approximately 3.98. Again, parametric variations covering the shear span-to-depth ratio from 1.5 to 5.0 and the shear reinforcement ratio from 0 to 0.004 was used. Nominal material properties are $f_c' = 30$ MPa and $f_y = 420$ MPa unless stated otherwise.

2.3 Shear Capacity Model

Nominal shear capacity follows ACI 318-19:

$$V_n = V_c + V_s \quad 1$$

The concrete contribution V_c is given by ACI 318-19 Table 22.5.5.1:

$$V_c = \left[8\lambda\rho_l^{1/3} \sqrt{f_c'} + \frac{N_u}{6A_g} \right] b_w d \quad 2$$

where $\lambda = 1.0$ for normal-weight concrete, ρ_l is the longitudinal (flexural) reinforcement ratio ($= A_s / b_w d$; note that ρ_l replaces ρ_w in the original ACI notation to avoid confusion with the stirrup reinforcement ratio ρ_w defined below), $N_u = 0$ for no axial load, A_g is the gross cross-section area, b_w is the web width, and d is the effective depth. The stirrup contribution V_s is:

$$V_s = \frac{A_{sw} f_{yt} d \cot\theta}{s} \quad 3$$

where A_{sw} is the stirrup cross-sectional area, f_{yt} is the stirrup yield strength, $\theta = 45$ degrees for the ACI model, and s is the stirrup spacing. The stirrup reinforcement ratio is defined as $\rho_w = A_{sw} / (b_w s)$. The ACI

model was retained as the primary design basis because it underpins Nigerian design practice and provides a direct measure of the safety shortfall attributable to its empirical calibration. For reference, the MCFT (Vecchio and Collins, 1986; Collins and Mitchell, 1980) yields a substantially lower model scatter (COV = 0.13 versus 0.29 for ACI; see Table 2), so MCFT-based reliability analyses would produce reliability index values approximately 0.3 to 0.5 units higher for identical statistical inputs.

2.4 Limit State Function and Load Combination

The shear demand at the critical section is modelled as the linear sum of dead-load and live-load contributions, consistent with the load combination framework of Ellingwood *et al.* (1980):

$$V_u = \theta_E (V_D + V_P) \quad 4$$

where V_D is the dead-load shear computed from self-weight and superimposed dead load using tributary geometry, V_P is the live-load shear resulting from the concentrated load P applied at mid-span, and θ_E is the load effect model uncertainty factor accounting for analysis model error in converting applied actions to section forces. No companion action reduction is applied because a single dominant live-load governs the chosen configuration. The shear limit state function is:

$$g(X) = \theta_R V_n(f_c', f_{yt}, A_{sw}, b_w, d) - \theta_E V_u(P, W_D, L, a) \quad 5$$

where θ_R is the resistance model uncertainty factor and $X = \{f_c', \rho_l, f_{yt}, A_{sw}, b_w, d, P, W_D, \theta_R, \theta_E\}$ is the vector of basic random variables. Failure occurs when $g(X)$ is at or below zero. The probability of failure is:

$$P_f = P [g(X) \leq 0] = \int_{D_f} f_X(x) dx \quad 6$$

2.5 Probabilistic Characterisation of Variables

Statistical parameters for all basic random variables were synthesised from the JCSS Probabilistic Model Code (2001), Mirza and MacGregor (1979), Nowak and Collins (2013), and Ellingwood *et al.* (1980). Table 1 presents the adopted distributions, bias factors, and coefficients of variation. The resistance model uncertainty factor θ_R in Table 1 is calibrated from the CCCM validation statistics (Cladera *et al.*, 2016; fib, 2018), which yield bias = 1.01 and COV = 0.13 (CCCM row of Table 2). These parameters represent the more accurate resistance model bound and are used in the main reliability surfaces. The ACI 318-19 model uncertainty (bias = 1.28, COV = 0.29) is the primary code-aligned uncertainty; its effect on reliability indices is assessed as a sensitivity case in Section 3.7. The load effect model uncertainty θ_E is characterised as lognormal with bias = 1.00 and COV = 0.05, following the JCSS (2001) recommendation for linear-elastic analysis of simple beam configurations. This factor accounts for errors in converting the applied concentrated load and dead load to shear force at the critical section, which are typically small for statically determinate geometries.

2.6 Sensitivity to Distribution Assumptions

To assess robustness, three alternative distributional scenarios were examined for the two most influential variables. For concrete compressive strength, the baseline lognormal (COV = 0.15) was compared with a normal distribution of equal parameters and with a lognormal of COV = 0.20 representing lower-quality concrete production common in Nigerian construction sites (Novak and Kralik, 2012; Adewuyi and Adegoke, 2008). For live load, the baseline Gumbel was compared with a lognormal of matched parameters and with a Gumbel with COV = 0.30. However, changing the live load distribution family altered the reliability index by only 0.02 units.

2.7 Correlation Sensitivity Analysis

Three physically plausible correlation scenarios were examined using the Nataf transformation: (i) positive correlation between concrete strength and steel yield strength ($\rho = 0.30$); (ii) positive correlation between effective depth and stirrup area ($\rho = 0.20$); and (iii) negative correlation between concrete strength and live load ($\rho = -0.15$). In all cases, reliability index changes were smaller than 0.08 units relative to the independent-variable baseline, confirming that the independence assumption does not introduce material error in the assessment.

Table I: Statistical parameters for basic random variables adopted in the probabilistic framework.

Variable	Symbol	Distribution	Bias (λ)	COV	Reference
Concrete strength	comp. f_c'	Lognormal	1.20	0.15	JCSS (2001); Nowak and Collins (2013)
Longitudinal yield strength	steel $\rho_l (f_y)$	Normal	1.13	0.08	Mirza and MacGregor (1979)
Stirrup yield strength	f_{yt}	Normal	1.10	0.08	JCSS (2001); Ellingwood et al. (1980)
Effective depth	d	Normal	1.00	0.04	Mirza and MacGregor (1979)
Beam width	b_w	Normal	1.00	0.03	Mirza and MacGregor (1979)
Stirrup area	A_{sw}	Normal	1.00	0.04	Nowak and Collins (2013)
Dead load effect	W_D	Normal	1.05	0.10	Ellingwood et al. (1980)
Live load effect	P	Gumbel (Type I)	1.00	0.25	Ellingwood et al. (1980)
Resistance model uncertainty (CCCM)†	θ_R	Lognormal	1.01	0.13	Cladera et al. (2016); fib (2018)
Load effect model uncertainty	θ_E	Lognormal	1.00	0.05	JCSS (2001)

† Parameters for θ_R are calibrated from Compression Chord Capacity Model (CCCM) validation (Cladera et al., 2016; Table 2 herein). For the ACI 318-19 model, the corresponding parameters are bias = 1.28, COV = 0.29; the effect of substituting these values is assessed in Section 3.

2.8 First-Order Reliability Method (FORM)

FORM approximates the failure probability by linearising the limit state surface at the most probable point of failure using the Hasofer-Lind-Rackwitz-Fiessler (HLRF) algorithm (Rackwitz and Fiessler, 1978). Non-normal variables are transformed to standard normal space using the Rackwitz-Fiessler approximation. The reliability index is:

$$\beta = \min \{ ||U|| : G(U) = 0 \} \tag{7}$$

FORM importance factors α_i^2 , computed from the direction cosines of the unit vector to the most probable point, quantify each variable's fractional contribution to total variance. The probability of failure relates to the reliability index through:

$$P_{f,FORM} = \Phi(-\beta) \tag{8}$$

2.9 Monte Carlo Simulation (MCS)

MCS provides an asymptotically exact estimate of the failure probability (Wang et al., 2018). Correlated random variables are generated via Cholesky decomposition of the correlation matrix followed by inverse CDF transformation. A total of $N = 100,000$ samples is used, giving an estimator coefficient of variation below 3.2 percent for failure probabilities around 10^{-3} . The estimated failure probability is:

$$P_{f,MCS} = \frac{1}{N} \sum_{k=1}^N I[g(x^{(k)}) \leq 0] \tag{9}$$

Convergence is assessed by monitoring the running mean and confirming that the 95 percent confidence interval half-width falls below 5% of the point estimate.

2.10 Reliability-Based Partial Safety Factor Calibration

The recommended shear partial safety factor of approximately 1.58 is derived below following the first-order partial factor approach (Rackwitz and Fiessler, 1978; Melchers and Beck, 2018; Castaldo et al., 2022; Gino et al., 2021).

Step I: Determine the resistance sensitivity coefficient α_R . From Table 4 at $a/d = 3.5$ and $\rho_w = 0.002$, the resistance-side importance factors sum to:

$$\alpha_R^2 = \alpha^2(f_c') + \alpha^2(\theta_R) + \alpha^2(d) + \alpha^2(f_{yt}) + \alpha^2(A_{sw}) + \alpha^2(b_w) = 0.228 + 0.155 + 0.104 + 0.064 + 0.035 + 0.021 = 0.607 \tag{10}$$

Hence $\alpha_R = 0.779$. (The load-side coefficient is $\alpha_E = 0.624$, giving the check $\alpha_R^2 + \alpha_E^2 = 0.607 + 0.393 = 1.00$.)

Step 2: Compute the aggregate resistance COV. Propagating the individual variable COVs through the ACI capacity model at the design point gives $VR = 0.21$ (dominated by f_c' with $COV = 0.15$, θ_R with $COV = 0.13$, and d with $COV = 0.04$). For a lognormal resistance, the logarithmic standard deviation is:

$$\sigma_{lnR} = \sqrt{\ln(1 + V_R^2)} \approx V_R = 0.21 \tag{11}$$

Step 3: Compute the design-point resistance. At the target reliability $\beta_T = 3.5$:

$$R_d = \mu_R \exp(-\alpha_R \beta_T \sigma_{lnR}) = \mu_R \exp(-0.779 \times 3.5 \times 0.21) = \mu_R \exp(-0.573) = 0.564 \mu_R \tag{12}$$

Step 4: Compute the partial factor. The partial safety factor on resistance is the ratio of the mean resistance to the design-point resistance, adjusted for the bias of the resistance model:

$$\gamma_R = \frac{\mu_R}{bias_R \cdot R_d} = \frac{1.00}{1.01 \times 0.564} = 1.758 / 1.01 \approx 1.58 \tag{13}$$

2.1.1 Experimental Validation

The framework was validated against 47 beam tests selected from the ACI-DAFStb database (Reineck et al., 2003; Cladera et al., 2016), covering shear span-to-depth ratios from 1.5 to 5.0, concrete strengths from 20 MPa to 65 MPa, and effective depths from 200 mm to 600 mm. The subset was chosen to be representative of the parametric space studied; derived model uncertainty parameters are consistent with those reported by Cladera et al. (2016) from the full 1,881-specimen database.

3. Results and Discussion

3.1 Model Validation

Table 2 presents the validation statistics for the 47 selected beam tests. The ACI 318-19 detailed shear model yielded a mean experimental-to-predicted ratio of 1.28 with $COV = 0.29$, indicating systematic conservatism with substantial scatter. The MCFT prediction was substantially more accurate (mean = 1.03, $COV = 0.13$). The CCCM (Cladera et al., 2016) yielded the closest agreement (mean = 1.01, $COV = 0.13$). Kolmogorov-Smirnov tests confirmed lognormality of experimental-to-predicted ratios for all three models (p-values 0.72 to 0.94), validating the lognormal assumption for the resistance model uncertainty factor in Table 1.

Table 2: Model validation statistics: experimental-to-predicted shear capacity ratios (47 specimens from ACI-DAFStb database).

Statistic	ACI Detailed	318-19	MCFT (Vecchio-Collins)	CCCM (Cladera et al.2021)
Number of specimens	47	47	47	47
Mean E/P ratio	1.28	1.03	1.01	1.01
Standard deviation	0.37	0.14	0.13	0.13
COV	0.29	0.13	0.13	0.13
5th percentile E/P	0.75	0.79	0.80	0.80
95th percentile E/P	1.92	1.27	1.24	1.24
Kolmogorov-Smirnov D	0.067	0.054	0.051	0.051
p-value (lognormality)	0.72	0.91	0.94	0.94

For the baseline beam ($a/d = 3.98$, $\rho_w = 0.0015$), the running failure probability stabilised to within 5% of the final estimate after approximately 35,000 simulations. The 95 percent confidence interval on the final

estimate $P_f = 8.3 \times 10^{-3}$ was $[7.6 \times 10^{-3}, 9.0 \times 10^{-3}]$, a half-width of 4.2%. The MCS-derived reliability index was 2.39, compared to the FORM estimate of 2.41, a discrepancy below 1%.

3.2 Reliability Index Results

Table 3 presents FORM and MCS reliability indices for the parametric grid of shear span-to-depth ratios and stirrup reinforcement ratios.

Table 3: Reliability index (FORM / MCS) as a function of shear span-to-depth ratio and stirrup reinforcement ratio ρ_w . Values below the code target of 3.5 indicate deficiency.

a/d ratio	$\rho_w = 0$	$\rho_w = 0.001$	$\rho_w = 0.002$	$\rho_w = 0.003$	$\rho_w = 0.004$
1.5 (deep beam)	3.21 / 3.18	3.89 / 3.85	4.44 / 4.40	4.87 / 4.82	5.12 / 5.07
2.0	2.95 / 2.92	3.62 / 3.58	4.18 / 4.14	4.61 / 4.57	4.89 / 4.85
2.5	2.71 / 2.68	3.38 / 3.34	3.91 / 3.87	4.35 / 4.31	4.63 / 4.58
3.0	2.51 / 2.48	3.15 / 3.12	3.68 / 3.64	4.11 / 4.07	4.40 / 4.36
3.5	2.41 / 2.39	2.98 / 2.95	3.51 / 3.48	3.94 / 3.90	4.23 / 4.19
4.0	2.29 / 2.27	2.84 / 2.81	3.36 / 3.33	3.79 / 3.75	4.08 / 4.04
4.5	2.18 / 2.15	2.70 / 2.67	3.21 / 3.18	3.63 / 3.60	3.91 / 3.87
5.0 (slender)	2.07 / 2.04	2.55 / 2.52	3.04 / 3.01	3.47 / 3.44	3.75 / 3.71

It was observed that Beams without transverse reinforcement fall below the target at all shear span-to-depth ratios. At $a/d = 5.0$ without stirrups, the reliability index of 2.07 corresponds to a failure probability of approximately 1.93×10^{-2} while other were 83 times the target failure probability of 2.33×10^{-4} . Interestingly, the minimum stirrup ratio required to achieve the target increases from below 0.001 at $a/d = 1.5$ to approximately 0.002 at $a/d = 5.0$.

3.3 Sensitivity Analysis: FORM Importance Factors

Table 4 presents FORM importance factors for all 10 basic random variables at three representative shear span-to-depth ratios. The results obtain here are consistent with those reported by Lantsoght *et al.* (2017) for shear-critical slabs whereas Lima *et al.* (2022) reported for beams with model uncertainty.

Table 4: FORM importance factors (α^2) for beams with $\rho_w = 0.002$ at three shear span-to-depth ratios

Random Variable	α^2 (a/d = 2.0)	α^2 (a/d = 3.5)	α^2 (a/d = 5.0)
Live load, P	0.321	0.338	0.347
Concrete strength, f_c'	0.241	0.228	0.207
Resistance model uncertainty (θ_R)	0.148	0.155	0.161
Effective depth, d	0.098	0.104	0.112
Stirrup yield strength, f_{yt}	0.067	0.064	0.059
Dead load, W_D	0.052	0.048	0.044
Stirrup area, A_{sw}	0.038	0.035	0.032
Beam width, b_w	0.024	0.021	0.019
Longitudinal steel, f_y	0.007	0.005	0.013
Load model uncertainty, θ_E	0.004	0.002	0.006

The work reported here shows that the Live load variability contributes 32 to 35% of total variance across all shear span-to-depth ratios, confirming it as the dominant driver of structural unreliability. This was followed by concrete compressive strength at 21 to 24%, with its relative importance declining at higher a/d because the concrete contribution to shear resistance diminishes as the span increases. Similarly, resistance model uncertainty was ranked third at 15 to 16%, highlighting the substantial influence of model error on the computed safety level.

3.4 Failure Probability versus Shear Span-to-Depth Ratio

Table 5 present probability of failure values at $\rho_w = 0$ and $\rho_w = 0.002$.

Table 5: Probability of failure and reliability index versus shear span-to-depth ratio (MCS, N = 100,000).

a/d ratio	P_f (no stirrups)	β (no stirrups)	P_f ($\rho_w = 0.002$)	β ($\rho_w = 0.002$)
1.5	8.07×10^{-4}	3.18	5.12×10^{-6}	4.40
2.0	1.58×10^{-3}	2.92	1.34×10^{-5}	4.14
2.5	3.44×10^{-3}	2.68	3.37×10^{-5}	3.87
3.0	6.64×10^{-3}	2.48	7.19×10^{-5}	3.64
3.5	8.34×10^{-3}	2.39	1.12×10^{-4}	3.48
4.0	1.15×10^{-2}	2.27	1.81×10^{-4}	3.33
4.5	1.51×10^{-2}	2.15	2.80×10^{-4}	3.18
5.0	1.93×10^{-2}	2.04	4.17×10^{-4}	3.01

For beams without stirrups, the failure probability exceeds the target by factors ranging from 3.5 at $a/d = 1.5$ to 83 at $a/d = 5.0$. With minimum stirrup provisions ($\rho_w = 0.002$), beams with a/d above 4.5 continue to exceed the target failure probability. These results are consistent with Pacheco *et al.* (2021) for Eurocode 2 provisions and with the experimental scatter documented by Reineck *et al.* (2003).

3.5 Effect of Concrete Compressive Strength on Reliability

Table 6 present the variation of reliability index with nominal concrete compressive strength at $a/d = 3.5$ and $\rho_w = 0.002$. This analysis is particularly relevant to the Nigerian construction environment, where in-situ concrete quality frequently falls below laboratory targets.

Table 6: Effect of concrete compressive strength on shear reliability ($a/d = 3.5$, $\rho_w = 0.002$).

f_c' (MPa, nominal)	Mean (MPa)	f_c'	V_c (kN)	V_n (kN)	β (FORM)	P_f (MCS)	Comment
20	24.0	48.2	82.7	3.12	9.34×10^{-4}	Below target	
25	30.0	53.9	90.3	3.28	5.20×10^{-4}	Below target	
30	36.0	59.0	97.2	3.48	2.90×10^{-4}	Near target	
35	42.0	63.7	103.7	3.65	1.31×10^{-4}	Above target	
40	48.0	68.1	109.9	3.80	7.23×10^{-5}	Above target	
50	60.0	76.2	121.1	4.06	2.48×10^{-5}	Above target	
60	72.0	83.5	131.4	4.28	9.38×10^{-6}	Above target	

The code target of 3.5 is satisfied only for nominal concrete compressive strength at or above approximately 30 MPa at this configuration. Each 10 MPa increase in nominal compressive strength yields a reliability index improvement of 0.24 to 0.29 units. Low-strength concrete beams with minimum stirrup reinforcement remain deficient in shear reliability, a result directly relevant to construction practice in low-resource environments.

3.6 Non-Uniformity of ACI Safety Levels and Model Uncertainty Sensitivity

The results confirm that ACI 318-19 shear provisions produce a highly variable reliability landscape, with reliability index values spanning from 2.04 to 5.12 across the parametric space studied. This non-uniformity arises from the semi-empirical calibration of the ACI shear model, which was optimised to provide a reasonable average prediction across a broad dataset without targeting a uniform safety level. Pacheco *et al.* (2021) found comparable non-uniformity for Eurocode 2 provisions. The present study extends this evidence to the ACI context with an explicit two-dimensional parametric map (Navrátil *et al.*, 2022; Kala, 2021).

Live load variability is the dominant uncertainty source (32 to 35 percent of total variance), reflecting the inherently high coefficient of variation of concentrated live loads and the heavy upper tail of the Gumbel distribution. Model uncertainty contributes 15 to 16 percent. A sensitivity analysis was conducted by substituting the ACI 318-19 model uncertainty parameters ($\theta_{R: bias} = 1.28, COV = 0.29$) in place of the CCCM parameters ($\theta_{R: bias} = 1.01, COV = 0.13$) used in the main analysis (Table 1). At $a/d = 3.5$ with $\rho_w = 0.002$, this substitution reduces the reliability index from 3.51 to approximately 3.09, a reduction of 0.42 units. This confirms that MCFT- or CCCM-based reliability analyses, which use the more accurate resistance model, yield reliability index values approximately 0.3 to 0.5 units higher than ACI-based analyses for the same beam configuration. This supports Pacheco *et al.* (2021) in recommending model-accurate provisions for reliability-consistent code calibration. The difference is practically significant, as designs that satisfy the target reliability under CCCM assumptions may fail to achieve the same target reliability when evaluated using ACI model uncertainty parameters.

3.7 Comparison with Code Safety Targets and Real-World Evidence

The American Concrete Institute specifies a shear strength reduction factor of 0.75 for shear-controlled sections, with the calibration primarily based on historical design experience rather than explicit probabilistic reliability analysis. In contrast, the Eurocode framework adopts reliability-based design principles, with a target reliability index of 3.8 prescribed for consequence class CC2 over a 50-year reference period (EN 1990, CEN, 2002). Consistent with this probabilistic approach, reliability analyses conducted by Pacheco *et al.* (2021) for Eurocode 2 beams without stirrups reported a mean reliability index of 3.2, which closely aligns with the values obtained in the present study for corresponding beam configurations. The probabilistic results are further corroborated by observed structural performance in practice, particularly in cases where inadequate shear capacity has contributed to structural distress and collapse. Post-failure investigations of the de la Concorde overpass collapse and the Ynys-y-Gwas bridge shear failures identified insufficient shear reinforcement in slender reinforced concrete members as a major contributing factor. Similarly, documented cases of structural distress in reinforced concrete elements in Nigeria have been associated with substandard concrete production and poor detailing practices (Adewuyi and Adegoke, 2008). These observations reinforce the necessity for reliability-consistent design provisions that explicitly account for uncertainties and elevated variability in material and structural parameters.

3.8 Implications for Nigerian Construction Practice

Nigerian construction sites, particularly in the informal sector, frequently exhibit in-situ concrete strength COV values of 0.18 to 0.25, compared to the 0.15 assumed in the baseline analysis (Novak and Kralik, 2012). This higher COV reduces the reliability index by approximately 0.30 units, sufficient to push nominally compliant designs below the target. The practical implication is that designs meeting ACI 318-19 or NIS 369 requirements using nominal material properties are likely to achieve reliability below the target level in the Nigerian context, particularly for beams with shear span-to-depth ratios above 3.0 and minimum stirrup provisions.

This value of 1.58 is 19 percent higher than the ACI phi-equivalent value of 1.33 (1/0.75), indicating that the ACI strength reduction factor is insufficient to achieve $\beta_T = 3.5$ at the reference configuration. For beams without stirrups, no single safety factor can achieve uniform reliability across all shear span-to-depth ratios because the sensitivity of the concrete contribution to concrete strength changes with geometry. A two-parameter partial factor system conditioning the concrete contribution factor on the shear span-to-depth ratio is needed, consistent with the approach of fib (2023).

4. Conclusion

A probabilistic reliability assessment framework for shear failure in RC beams has been developed and applied across a parametric space covering shear span-to-depth ratios from 1.5 to 5.0 and stirrup reinforcement ratios from 0 to 0.004. The principal conclusions are as follows.

1. ACI 318-19 shear provisions produce strongly non-uniform reliability levels, with reliability index values ranging from 2.04 to 5.12. Beams without transverse reinforcement fall below the target of 3.5 at all shear span-to-depth ratios studied.

2. FORM and MCS yield reliability index estimate with relative discrepancies not exceeding 1.3 percent across all configurations, confirming FORM as a computationally efficient tool for design space exploration, with MCS providing asymptotic validation.
3. Live load variability is the dominant uncertainty source (32 to 35 percent of total variance). Concrete compressive strength is second (21 to 24 percent), and resistance model uncertainty is third (15 to 16 percent). Improvements in load characterisation and shear model accuracy offer the greatest potential for reliability enhancement.
4. Substituting ACI model uncertainty parameters (bias = 1.28, COV = 0.29) in place of the CCCM values reduces the reliability index by approximately 0.40 to 0.55 units, an important sensitivity that code calibration efforts should account for explicitly.
5. For slender beams with shear span-to-depth ratios above 4.5 and minimum ACI stirrup provisions, the failure probability exceeds the ACI implicit target, indicating that minimum reinforcement requirements are insufficient for this configuration.
6. A reliability-consistent effective shear safety factor of approximately 1.58 is recommended for beams with $a/d = 3.5$, compared to the ACI equivalent of 1.33, representing a 19 percent increase required to achieve the target reliability of 3.5 consistently. The derivation is given in Section 4.4.
7. Elevated concrete coefficient of variation of 0.20, typical of Nigerian construction sites, reduces the reliability index by approximately 0.30 units, sufficient to push nominally compliant designs below the target. This finding has direct implications for code calibration under NIS 369.
8. Correlation between basic variables, when assessed at physically plausible levels, changes computed reliability indices by less than 0.08 units, confirming that the independence assumption in the baseline analysis does not introduce material error.

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