



ORIGINAL RESEARCH ARTICLE

**BEST-FIT PROBABILITY DISTRIBUTION MODEL FOR ANNUAL MONTHLY
MAXIMUM RAINFALL IN CALABAR CITY, NIGERIA**

G. D. Akpen, I. D. Ijasini and G. N. Ekwo

Department of Civil Engineering, Joseph Sarwuan Tarka University, Makurdi, Benue State, Nigeria

*Corresponding author's email address: akpen.gabriel@uam.edu.ng

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ABSTRACT

The choice of a suitable model that could predict the possibility of occurrence of rainfall event of a specified magnitude depends mainly on the characteristics of rainfall data at a particular site. This study was aimed at determining the best-fit probability distribution model for annual maximum monthly rainfall. Fifty years rainfall data (1969-2018) obtained from Nigerian Meteorological Agency (NIMET), Calabar, were collated to form an annual series ranked in a decreasing order of magnitude. The data were then evaluated with 6 probability distributions namely; Normal, Log-Normal, Pearson Type III, Log-Pearson Type III, Gumbel Extreme Value Type I and Log-Gumbel probability distributions. The best-fit probability distribution model at the study area was selected based on the results of 4 goodness of fit tests namely; Mean Absolute Deviation Index (MADI), Chi-Squared(χ^2) test, Root Mean Square Error (RMSE) and Relative Root Mean Square Error (RRMSE). The performance of the 6 distributions were ranked and the total scores of 8, 23, 16, 12, 21 and 4 were allocated to Normal, Log-Normal, Pearson Type III, Log-Pearson Type III, Gumbel Extreme Value Type I and Log-Gumbel probability distributions, respectively. The results indicate that the best-fit probability distribution model was Log-Normal, which was used to predict rainfall values of 566.8, 664.0, 721.6, 788.1, 834.4, 878.4, 920.7, 974.8 and 1014.4 mm for different return periods of 2, 5, 10, 25, 50, 100, 200, 500 and 1000 years, respectively. It is, therefore, recommended that Log-Normal distribution is the preferred model for frequency analysis of rainfall data for the planning and design of hydraulic structures in the study area. This is necessary for effective flood prevention and mitigation interventions.

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1.0 Introduction

Rainfall is one of the most widely researched climatic variables in terms of its absence or presence, extremity or scarcity, stability or variability, as well as its vulnerability or essentiality (Oruonye, 2012; Gabriel et al., 2018; Hassanyar et al., 2018 and Aho et al., 2019a). Rainfall is a major source of fresh water replenishment for planet earth. However, too much or too little can mean the difference between prosperity and disaster (Oyegoke et al., 2017). As a random hydrological event, the occurrence of rainfall cannot be predicted with certainty because its temporal and spatial distribution is very complex and irregular. This uncertainty is further aggravated by the current global problem of climate change. However, it is possible to use rainfall data spanning over a long period of time to estimate the likelihood of a rainfall of a particular magnitude

occurring within a specified period of time using frequency analysis. According to Izinyon and Ajumuka (2013) and Win and Win (2014), there is no universally accepted frequency distribution model for rainfall frequency analysis. Few probability distribution functions have been tested and adopted locally, while some others are peculiar to hydrologic characteristics of other regions.

The ability to predict the possibility of occurrence of a particular rainfall magnitude is one of the preliminary stages of storm water drainage design that can help individuals, authorities and engineers to plan for extreme eventualities such as flood, drought, landslides, and thunderstorms, among others (Oyegoke *et al.*, 2017). Tao *et al.* (2002) stated that several probability models have been developed to describe the distribution of annual extreme rainfalls at a single site. However, the choice of a suitable model is still one of the major problems in engineering practice since there is no general agreement as to which distribution(s) that should be used for the frequency analysis of extreme rainfalls.

Khudri *et al.* (2013) reported that the generalized extreme value and generalized gamma four parameter (4P) distributions provide the best fit for 50% of the data from rainfall gauging stations studied in Bangladesh. Olofintoye *et al.* (2009) showed that for peak daily rainfalls of selected cities in Nigeria, the Log-Pearson type III (LP3) distribution performed best by occupying 50% of the total stations, while Pearson type III (P3) performed second best by occupying 40% of the total stations and lastly by Log-Gumbel occupying 10% of the total stations. Aho *et al.* (2019b) carried out frequency analysis and reported that the best fit probability distribution function (PDF) for annual and partial series rainfall data in Makurdi metropolis is generalized extreme value (GEV) distribution and Generalized Pareto (GPA) distribution respectively.

The selection of an appropriate model depends mainly on the characteristics of available rainfall data at a particular site. Hence, it is necessary to evaluate many available distributions in order to find a suitable model that could provide accurate extreme rainfall estimates. More so, recent devastation caused by flood in different parts of the world in addition to the challenges currently posed by uncertainties occasioned by climate change phenomenon has made the reliability in estimation of rainfall events more imperative (Akpen *et al.*, 2019). The value of such studies especially, in this era of failure/collapse of bridges, culverts, dams and other drainage structures leading to loss of means of livelihoods and sometimes, loss of lives cannot be over emphasized.

The objective of this study is to determine the best-fit probability distribution model for annual monthly maximum rainfall over Calabar city. This is required for planning and design of effective flood prevention, control and mitigation measures.

2.0 MATERIALS AND METHODS

2.1 Study Area

Calabar is the capital of Cross River State, Nigeria. The city is adjacent to the Calabar and Great Kwa rivers and creeks of the Cross River (from its inland delta). It lies between latitudes $05^{\circ} 45'$

30" N and 05° 08' 30" N and longitudes 8° 11' 21" E and 8° 30' 00" E (Figure 1). Calabar is often described as the tourism capital of Nigeria. Administratively, the city is divided into Calabar Municipal and Calabar South Local Government Areas. It has an area of 406 square kilometers and a population of 371, 022 as at 2006 Census (Falola and Warnock, 2007; Ogarekpe, 2014; Ottong et al., 2010). Calabar has a tropical monsoon climate with a lengthy wet season spanning 10 months and a short dry season covering the remaining 2 months each year. The harmattan which significantly influences weather in West Africa is noticeably less pronounced in the city. Temperatures are relatively constant throughout the year with average high temperatures usually ranging from 25 to 28 degrees Celsius. There is only little variance between day time and night time temperatures, with fewer degrees lower during nights. In some wet years (1969, 1976, 1980, 1995, 1996, 1997, 1999, 2001, 2005, 2007, 2008, 2010 – 2015, 2017 and 2018), annual rainfall depths have been observed to rise over 3000 mm and above (Ogarekpe, 2014; Antigha, 2012).

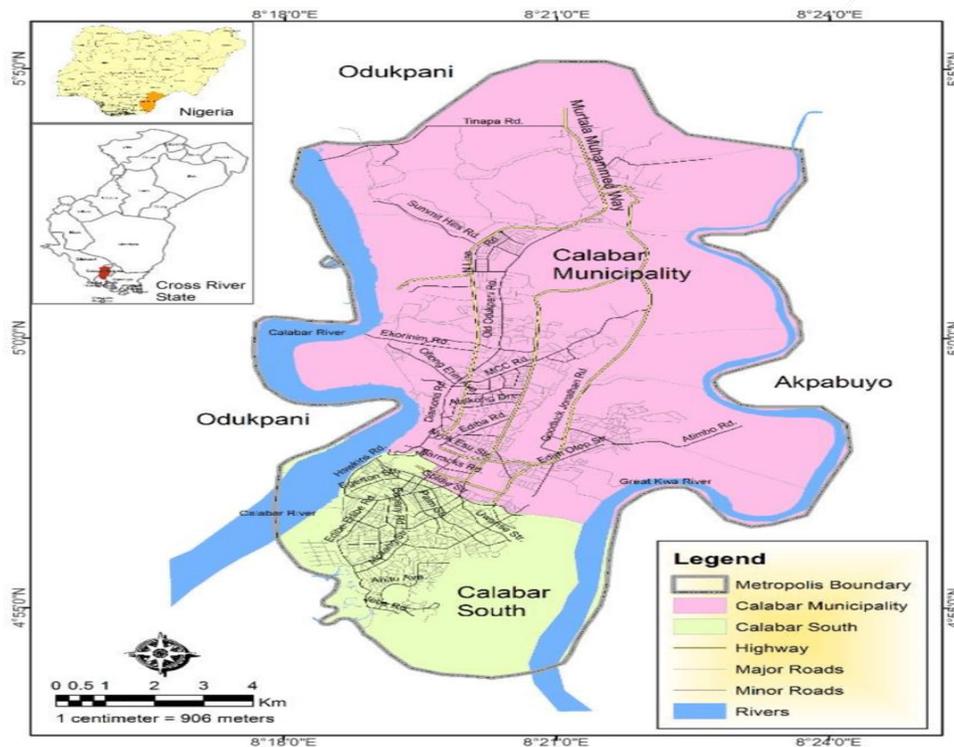


Figure 1: Map of Calabar, Nigeria (Source: Erhabor et al., 2019)

2.2 Data Collection and Preparation

The monthly rainfall data for 50 years (January, 1969 to December, 2018) was obtained from the Nigerian meteorological Agency (NIMET), Calabar, Nigeria. The data was sorted according to years and months. The monthly maximum rainfall depths for 50 years of record were collated to form an annual series which were ranked in decreasing order of magnitude with their corresponding years and months as shown in Table I. The ranked annual monthly maximum series of rainfall values were then analyzed.

The probability of an event being equaled or exceeded in any one year was calculated using the Equation 1 (Weibull, 1939), while the recurrence interval, which is the inverse of the probability of its occurrence is given by Equation 2.

$$\text{Probability of occurrence; } P_{ro} = \frac{m}{N+1} \quad (1)$$

$$\text{Return period; } T_r = \frac{N+1}{m} \quad (2)$$

Where m is the rank number and N is the total number of observed data

Table 1: Annual Monthly Maximum Rainfall, Probability and Return Period in Calabar (1969 - 2018)

Year	Month	x_i (mm)	Rank (m)	P_{ro} (%)	T_r (years)	Year	Month	x_i (mm)	Rank (m)	P_{ro} (%)	T_r (years)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
2012	Aug.	861.3	1	1.96	51.0	2009	July	577.4	25	49.02	2.0
2005	July	828.2	2	3.92	25.5	1975	July	552.6	26	50.98	1.96
1997	July	796.6	3	5.88	17.0	1988	Sept.	538.3	27	52.94	1.9
1980	Aug.	728.7	4	7.84	12.8	1986	July	533.8	28	54.90	1.8
1969	Aug.	726.9	5	9.80	10.2	1976	Aug.	526.8	29	56.86	1.76
1978	Sept.	721.2	6	11.76	8.5	1985	May	520.5	30	58.82	1.7
2014	July	714.4	7	13.73	7.3	1981	July	519.7	31	60.78	1.6
1990	July	702.7	8	15.69	6.4	1972	July	510.8	32	62.75	1.59
2015	June	678.7	9	17.65	5.7	1982	July	505.7	33	64.71	1.55
1971	July	661.4	10	19.61	5.1	1991	Aug.	505.6	34	66.67	1.5
2018	Aug.	650.6	11	21.57	4.6	1983	June	504.6	35	68.63	1.46
2011	July	648.6	12	23.53	4.2	1998	June	504.5	36	70.59	1.4
1995	July	632.4	13	25.49	3.9	2013	May	499.9	37	72.55	1.38
1989	July	626.5	14	27.45	3.6	1999	Aug.	494.5	38	74.51	1.34
2002	Aug.	623.5	15	29.41	3.4	1987	Aug.	493.8	39	76.47	1.3
1996	Sept.	615.2	16	31.37	3.2	2001	May	491.4	40	78.43	1.28
2017	Aug.	611.4	17	33.33	3.0	2006	July	484.9	41	80.39	1.24
2010	June	611.3	18	35.29	2.8	1992	Sept.	481.4	42	82.35	1.21
1994	July	609.5	19	37.25	2.7	1993	Aug.	479.0	43	84.31	1.2
1979	June	599.3	20	39.22	2.5	2016	July	454.6	44	86.27	1.16
2008	July	597.7	21	41.18	2.4	1970	July	440.8	45	88.24	1.13
1977	July	597.6	22	43.14	2.3	1984	June	437.3	46	90.20	1.11
2000	July	597.6	22	43.14	2.3	2003	Sept.	399.2	47	92.16	1.09
1974	Sept.	588.4	23	45.10	2.2	2004	Aug.	391.9	48	94.12	1.06
2007	May	583.5	24	47.06	2.1	1973	Aug.	373.3	49	96.08	1.04

KEY: T_r = Return period, P_{ro} = Probability of occurrence, x_i = Rainfall depth

2.3 Descriptive Statistics of Annual Monthly Maximum Rainfall Series

The Arithmetic Mean (\bar{X}), Standard Deviation (σ), Coefficient of Variation (C_v), The coefficient of skewness (C_s) and coefficient of Kurtosis (C_k) were calculated using Equations 3-7 respectively.

$$\bar{X} = \frac{X_1+X_2+X_3+\dots+X_N}{N} = \frac{\sum_{i=1}^N X_i}{N} \tag{3}$$

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (X_i - \bar{X})^2} \tag{4}$$

$$C_v = \frac{\sigma}{\bar{X}} \tag{5}$$

$$C_s = \frac{m_3}{\sigma^3} \text{ or } \frac{N \sum_{i=1}^N (X_i - \bar{X})^3}{(N-1)(N-2)\sigma^3} \text{ given by Spiegel et al. (2011)} \tag{6}$$

$$C_k = \frac{m_4}{\sigma^4} \text{ or } \frac{N \sum_{i=1}^N (X_i - \bar{X})^4}{(N-1)(N-2)\sigma^4} \text{ given by Spiegel et al. (2011)} \tag{7}$$

Where, N = total number of observed data, 50 in this case,
 $X_1, X_2, X_3, \dots, X_N$ = monthly maximum rainfall data for each year,
 X_i = observed monthly maximum rainfall data for a specified year
 i = specified year (i ranged from 1-50),
 m_3 = Third moment about the mean of the set of data, and
 m_4 = Fourth moment about the mean of the set of data.

Table 2: Logarithms of Annual Monthly Maximum Rainfall in Calabar (1969 -2018)

Year	Month	Rainfall Depth	Rank	y_i	Year	Month	Rainfall Depth	Rank	y_i
(1)	(2)	(X_i) (mm)	(m)	= $\log_{10} X_i$	(1)	(2)	(X_i) (mm)	(m)	= $\log_{10} X_i$
2012	Aug.	861.3	1	2.9352	2009	July	577.4	25	2.7615
2005	July	828.2	2	2.9181	1975	July	552.6	26	2.7424
1997	July	796.6	3	2.9012	1988	Sept.	538.3	27	2.7310
1980	Aug.	728.7	4	2.8625	1986	July	533.8	28	2.7274
1969	Aug.	726.9	5	2.8615	1976	Aug.	526.8	29	2.7216
1978	Sept.	721.2	6	2.8581	1985	May	520.5	30	2.7164
2014	July	714.4	7	2.8539	1981	July	519.7	31	2.7158
1990	July	702.7	8	2.8468	1972	July	510.8	32	2.7083
2015	June	678.7	9	2.8317	1982	July	505.7	33	2.7039
1971	July	661.4	10	2.8205	1991	Aug.	505.6	34	2.7038
2018	Aug.	650.6	11	2.8133	1983	June	504.6	35	2.7029
2011	July	648.6	12	2.8120	1998	June	504.5	36	2.7029
1995	July	632.4	13	2.8010	2013	May	499.9	37	2.6989
1989	July	626.5	14	2.7969	1999	Aug.	494.5	38	2.6942
2002	Aug.	623.5	15	2.7948	1987	Aug.	493.8	39	2.6936
1996	Sept.	615.2	16	2.7890	2001	May	491.4	40	2.6914
2017	Aug.	611.4	17	2.7863	2006	July	484.9	41	2.6857
2010	June	611.3	18	2.7863	1992	Sept.	481.4	42	2.6825
1994	July	609.5	19	2.7850	1993	Aug.	479.0	43	2.6803
1979	June	599.3	20	2.7776	2016	July	454.6	44	2.6576
2008	July	597.7	21	2.7765	1970	July	440.8	45	2.6442

1977	July	597.6	22	2.7764	1984	June	437.3	46	2.6408	
2000	July	597.6	22	2.7764	2003	Sept.	399.2	47	2.6012	
1974	Sept.	588.4	23	2.7697	2004	Aug.	391.9	48	2.5932	
2007	May	583.5	24	2.7660	1973	Aug.	373.3	49	2.5721	
							$\sum X_i$	28,835.5	$\sum y_i$	137.6703

The logarithms of all the annual monthly maximum rainfall depths were calculated using Equation 8 as shown in column (5) of Table 2, thus:

$$y_i = \text{Log}x_i \quad (8)$$

Where, y_i = logarithmically transformed individual annual monthly maximum value of rainfall.

2.4 Determination of Best-Fit Probability Distribution Model

The ranked data were evaluated with 6 probability distribution models namely: Normal (N), Log-Normal (LN), Pearson Type III (P_3), Log-Pearson Type III (P_3), Gumbel Extreme Value Type I (EVI) and Log-Gumbel (LG) probability distribution models. The best fit model was selected based on the results of goodness of fit tests and the application of a scoring and ranking scheme. Subsequently, the best fit model of the study area was used to predict rainfall depths for return periods between 2 and 1000 years.

2.4.1 Normal Probability Distribution Model

The Normal distribution is also called Gaussian distribution or Normal Error Curve. The parameters of the distribution are mean and standard deviation while the Skewness is taken as zero. The estimation of rainfall (X_T), for a given return period (T_r) was calculated using the following procedure:

The mean (\bar{x}) of the observation was calculated using Equation 3, while the standard deviation (σ) was found using Equation 4.

The standard normal variate (Z), corresponding to an exceedence probability, P ($P = \frac{1}{T_r}$) was calculated by finding the value of an intermediate variable W :

$$W = \left[\ln \left(\frac{1}{P_2} \right) \right]^{\frac{1}{2}} \quad (0 < P \leq 0.5) \quad (9)$$

The standard normal variate, z (which is the same as the value of frequency factor K_T) that depends on return period for the Normal distribution was then found using Equation 10 (Chow et al., 1988):

$$Z = w - \frac{2.515517 + 0.802853W + 0.010328W^2}{1 + 1.432788W + 0.189269W^2 + 0.001308W^3} \quad (10)$$

When $P > 0.5$; $1 - P$ is substituted for P in Equation 9.

The estimated rainfall (X_T) was calculated from Equation 11 (Chow et al., 1988):

$$X_T = \bar{x} + Z\sigma = \bar{x} + K_T\sigma \quad (11)$$

2.4.2 Log-Normal (LN) Probability Distribution Model

A random variable has a Log-normal distribution, if the log of the random variable has a normal distribution. The procedure adopted here was similar to that of Normal distribution except that it was applied to the logarithmically transformed observed rainfall depths.

2.4.3 Pearson Type III (P₃) Distribution Model

The Pearson Type III distribution has 3 parameters and is bound in the left with positive Skewness. The 3 parameters of the distribution are mean, standard deviation and skewness. The frequency factor depends on the return period and the coefficient of skewness, C_S. When C_S = 0, the frequency factor is equal to the standard normal variate Z. When C_S ≠ 0, the frequency factor (K_T) was calculated using Equation 12 (Kite, 1977) expressed as:

$$K_T = Z + (Z^2 - 1)K + \frac{1}{3}(Z^3 - 6Z)k^2 - (Z^2 - 1)K^3 + ZK^4 + \frac{1}{3}K^3 \quad (12)$$

where, $K = \frac{C_S}{6}$.

2.4.4 Log Pearson Type III (LP₃) Probability Distribution Model

The mean (\bar{y}), standard deviation (σ^*) and coefficient of skewness (C_S^{*}) of the logarithmically transformed data were computed and the Pearson frequency factor (K_T) was found from Equation 12.

2.4.5 Gumbel Extreme Value Type I (EVI) Probability Distribution Model

The frequency factor (K_T) for EVI was calculated from Equation 13 (Chow, 1953) and the estimated rainfall, X_T was then calculated using Equation 11.

$$K_T = -\frac{\sqrt{6}}{\pi} \{0.5772 + \ln[\ln(\frac{T_r}{T_r-1})]\} \quad (13)$$

2.4.6 Log-Gumbel (LG) Probability Distribution Model

The mean (\bar{y}), standard deviation (σ^*) and coefficient of skewness (C_S^{*}) of the logarithmically transformed data were determined and the frequency factor (K_T) was computed from Equation 13.

For LN, LP₃, and LG distributions, the actual estimated rainfall (X_T), corresponding to log transformed estimated rainfall (Y_T), was computed using Equation 14 thus:

$$X_T = 10^{Y_T} \quad (14)$$

2.5 Goodness of Fit Test

In order to check the adequacy of fit of the observed rainfall data to the six probability distribution models; four goodness of fit tests namely; Mean Absolute Deviation Index (MADI), Chi-squared(x²), Root Mean Square Error (RMSE) and Relative Root Mean Square Error (RRMSE) tests were adopted.

2.5.1 Mean Absolute Deviation Index (MADI)

The mean absolute deviation index is the average distance between each data value and the mean. It was computed using the formula suggested by Agbonaye and Izinyon (2017):

$$MADI = \frac{1}{N} \sum_{j=1}^N \left| \frac{X_j - X_{T_j}}{X_j} \right| \quad (15)$$

Where, X_j = The observed value
 X_{T_j} = The value forecasted by the distribution
 N = The number of data points

The smaller the value of MADI obtained for a distribution, the more fitted it is to the observed data (Agbonaye and Izinyon, 2017).

2.5.2 Chi-Squared (χ^2) Test

This is a measure of the discrepancy existing between the observed and expected frequencies and is given by:

$$\chi^2 = \frac{(X_1 - X_{T_1})^2}{X_{T_1}} + \frac{(X_2 - X_{T_2})^2}{X_{T_2}} + \frac{(X_k - X_{T_k})^2}{X_{T_k}} = \sum_{j=1}^k \frac{(X_j - X_{T_j})^2}{X_{T_j}} \quad (16)$$

Where, X_j = Observed frequency
 X_{T_j} = Expected frequency
 j = Number of observations (1, 2, ... k)

The smaller the value of χ^2 for a distribution, the more fitted it is to the observed data. The test was performed at 5% and 1% levels of significance.

2.5.3 Root Mean Square Error (RMSE)

The root mean square error (RMSE) also known as the standard error is the sum of squares of the differences between observed and computed values. However, it was used to measure the difference between values predicted by a model and the observed values. These individual differences are called residuals. The RMSE was obtained using the formula suggested by Tao *et al.* (2002):

$$RMSE = \sqrt{\left(\frac{\sum (X_j - X_{T_j})^2}{(n-m)} \right)} \quad (23)$$

Where, X_j ($j = 1, 2, \dots, n$) are observed values
 X_{T_j} ($j = 1, 2, \dots, n$) are the corresponding values computed from the assumed probability distribution,
 n is the number of observations and;
 m is the number of parameters estimated for the distribution.

The smaller the value of RMSE obtained for a distribution, the more fitted it is to the observed data (Agbonaye and Izinyon, 2017).

2.5.4 Relative Root Mean Square Error (RRMSE)

The relative root mean square error was computed by dividing the root mean square error by the mean of observed data. The relative root mean square error provides a good picture of the overall fit of a distribution. It computes each error in proportion to the size of observation, thereby reducing the effect of outliers which are commonly found in hydrological data. The RRMSE was computed using the formula by Tao et al. (2002):

$$RRMSE = \sqrt{\frac{\sum \left(\frac{X_j - X_{Tj}}{X_j} \right)^2}{(n-m)}} \quad (24)$$

Where, $X_j (j = 1, 2, \dots, n)$ = observed values

$X_{Tj} (j = 1, 2, \dots, n)$ = the corresponding values computed from the assumed probability distribution,

n = number of observations, and;

m = the number of parameters estimated for the distribution.

The smaller the value of RRMSE obtained for a distribution, the more fitted it is to the observed data (Agbonaye and Izinyon, 2017).

2.6 Scoring and Ranking Scheme

Based on the results of the goodness of fit tests, a scoring scheme in which the best performing distribution with respect to a test criterion was assigned a score of 6, the next test was assigned 5 and the worst test assigned 1. The distribution with the highest total score in the study area based on the goodness of fit criteria was adjudged as the best distribution model for prediction of annual monthly maximum rainfall in the study area.

3. Results and Discussion

3.1 Annual Monthly Maximum Rainfall

The annual monthly maximum rainfall depths ranged from 373.3 mm to 861.3 mm, indicating a very large range of fluctuation during the study period (Figure 2). The minimum and maximum rainfall values were in the month of August, 1973 and 2012 respectively. Incidentally, incidences of severe flooding were reported in Calabar in the year, 2012, the year with the maximum monthly rainfall.

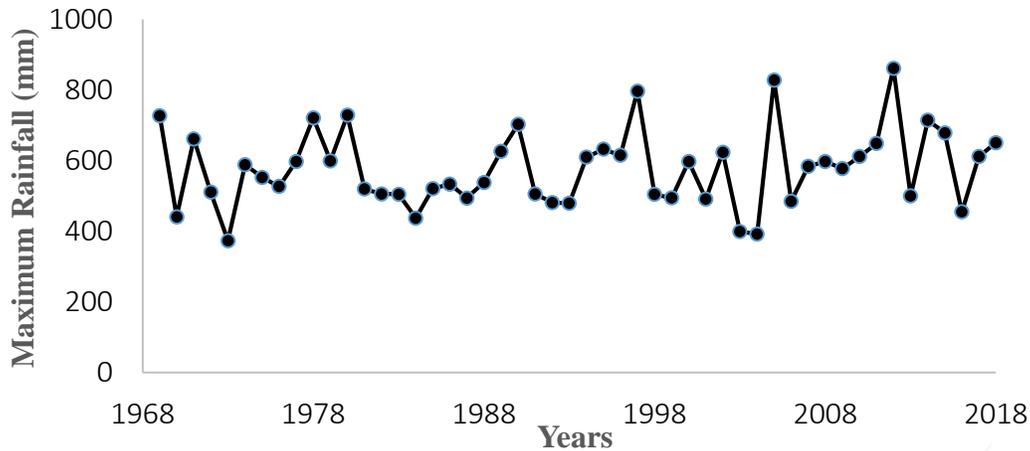


Figure 2: Annual Monthly Maximum Rainfall (mm) for Calabar (1969 - 2018)

The computed frequency factor (K_T) values and the estimated rainfall (X_T) values for different return periods of 2, 5, 10, 25, 50, 100, 200, 500 and 1000 years, are presented in Table 3. The results of the goodness of fit tests for the 6 distributions at the station were also presented in Table 4. Ranking the total scores, a summary of performance of the 6 probability distribution models was obtained as shown in Table 5. The Log-Normal distribution model, with a total score of 23 is adjudged the best fit model for the study area. However, Olofintoye *et al.* (2009) found Log-Pearson Type III probability distribution model as the best-fit model for the peak daily rainfall in Calabar. It follows that the probability distribution model used for monthly maximum rainfall in a given location may not necessary fit daily maximum rainfall for the same location. The Log-Normal model was then used to predict rainfall values shown in Table 2 for selected return periods of 2, 5, 10, 25, 50, 100, 200, and 500 and 1000 years. These rainfall estimates can provide useful guidance for policy makers and designers.

Table 3: Computed Values of Frequency Factor (K_T) and Rainfall Estimates (X_T) given by Six (6) Probability Distributions for different Return Periods for Calabar

Return Period (years)	Normal (N)		Log-Normal (LN)		Pearson Type III (P_3)		Log-Pearson Type III (LP_3)		Gumbel (EVI)		Log-Gumbel (EVI)	
	K_T	X_T (mm)	K_T	X_T (mm)	K_T	X_T (mm)	K_T	X_T (mm)	K_T	X_T (mm)	K_T	X_T (mm)
2	0.0000	576.7	0.0000	566.8	- 0.0878	567.1	0.0162	568.5	- 0.1642	558.8	-0.1642	549.5
5	0.8414	668.4	0.8414	664.0	0.8043	664.4	0.8456	664.7	0.7195	655.2	0.7195	649.1
10	1.2818	716.5	1.2818	721.6	1.3238	721.0	1.2697	719.9	1.3041	718.9	1.3041	724.6
25	1.7511	767.6	1.7511	788.1	1.9193	786.0	1.7161	783.1	2.0421	799.4	2.0421	832.5
50	2.0541	800.7	2.0541	834.4	2.3226	830.0	2.0015	826.2	2.5921	859.3	2.5921	923.4
100	2.3268	830.4	2.3268	878.4	2.7131	872.5	2.2552	866.8	3.1402	919.1	3.1402	1024.0
200	2.5762	857.6	2.5762	920.7	3.0757	912.1	2.4851	905.1	3.6806	978.0	3.6806	1133.7
500	2.8785	890.6	2.8785	974.8	3.5355	962.2	2.7612	953.5	4.3949	1055.9	4.3949	1296.9
1000	3.0905	913.7	3.0905	1014.4	3.8610	997.7	2.9531	988.6	4.9353	1114.8	4.9353	1435.8

Table 4: Goodness-of-Fit (GoF) Test Results for the Distributions at Calabar

Station	GoF Criteria	Test	Probability Distribution Models					
			N	LN	P_3	LP_3	EVI	LG
Calabar	MADI		0.0099	0.0030	0.0040	0.0048	0.0029	0.0214
	χ^2		57.1018	4.7976	8.7384	12.0788	7.3030	219.2021
	RMSE		84.8856	25.5417	37.1754	43.5557	32.3466	201.4082
	RRMSE		0.0854	0.0261	0.0376	0.0439	0.0339	0.2001

For the chi-squared (χ^2) test:

At $\alpha = 0.05$, degree of freedom = 8, and the critical value = 15.5

At $\alpha = 0.01$, degree of freedom = 8, and the critical value = 20.1

Table 5: Scoring and Ranking Scheme of the Statistical Tests Results for the Distributions at Calabar

STATION	PROBABILITY DISTRIBUTION MODELS	GOODNESS-OF-FIT TEST CRITERIA				TOTAL SCORE	RANK
		MADI	χ^2	RMSE	RRMSE		
CALABAR	N	2	2	2	2	8	5 TH
	LN	5	6	6	6	23	1 ST
	P3	4	4	4	4	16	3 RD
	LP3	3	3	3	3	12	4 TH
	EVI	6	5	5	5	21	2 ND
	LG	1	1	1	1	4	6 TH

Table 6: Estimated Rainfall (X_T) Values (mm) for various Return Periods given by the Best Fit Model for Calabar

Station	Best-Fit Model	Return Period (Years)								
		2	5	10	25	50	100	200	500	1000
Calabar	Log-Normal	566.8	664.0	721.6	788.1	834.4	878.4	920.7	974.8	1014.4

3.2 Rainfall Frequency Curve

The annual maximum rainfall estimates for the best fit probability distribution model in Table 6, were used to develop the rainfall frequency curve as presented in Figure 3.

More so, the yearly wise annual maximum monthly rainfall depths ranging from 373.3 mm to 861.3 mm during the study period are also presented in the Figure 3. The time series graph is important for showing how data changes over time.

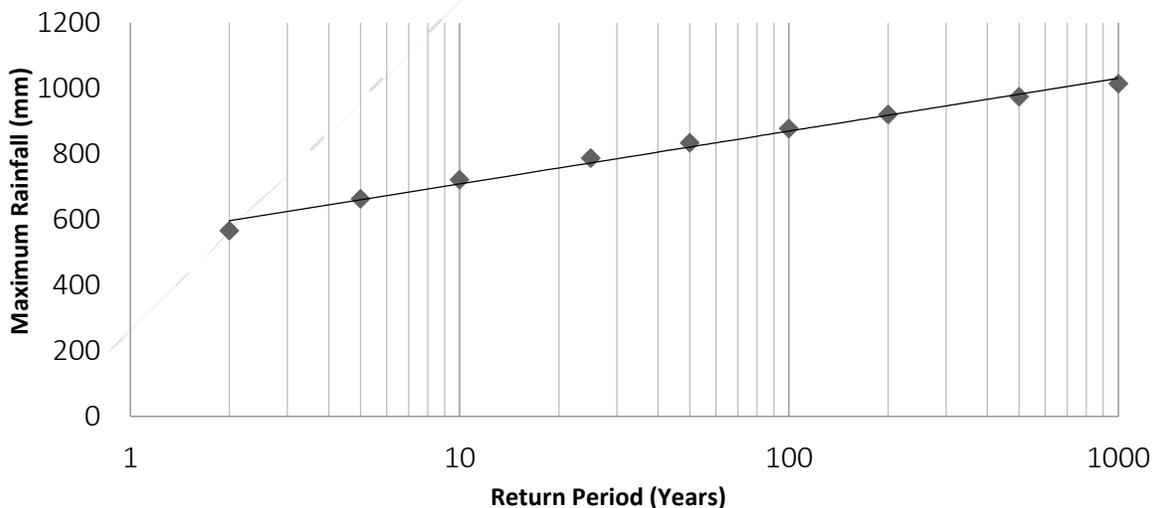


Figure 3: Rainfall Frequency Curve for Calabar (based on Log-Normal distribution model)

4. Conclusion

Accurate extreme rainfall estimates provided by a suitable model are needed by individuals, authorities and engineers in their planning and designing of hydraulic structures. The results of

this study, reveals that, Log-Normal distribution was the best-fit model for Calabar in the case of annual maximum monthly rainfall data. The rainfall frequency curve (Figure 3) shows the predicted rainfall values that were obtained from the best-fit model, which may serve as a guide in hydrological designs. This study has provided useful engineering design tool for planning and designing of hydraulic structures and other flood control and mitigation measures. It is recommended that the developed frequency curve (based on the best fit model) should be adopted for prediction of rainfall amount for design of hydraulic structures in the study area.

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