ARID ZONE JOURNAL OF ENGINEERING, TECHNOLOGY & ENVIRONMENT
AZOJETE September 2022. Vol. 18(3):517-526
Published by the Faculty of Engineering, University of Maiduguri, Maiduguri, Nigeria.
Print ISSN: 1596-2644, Electronic ISSN: 2545-5818
www.azojete.com.ng

ORIGINAL RESEARCH ARTICLE

ANALYTICAL DETERMINATION OF QUEUEING SYSTEM PERFORMANCE FOR SUSTAINABLE ECONOMIC DEVELOPMENT

P. B. Anyin[1*], and A. A. Etika[2]
[1*]*Department of Civil Engineering, University of Calabar, Calabar, Nigeria*
[2]*Department of Civil Engineering, Cross River University of Technology, Calabar, Nigeria*
**Corresponding author's email address: profpeteranyin@unical.edu.ng*

ARTICLE INFORMATION

Submitted 30 Dec., 2022
Revised 24 April, 2022
Accepted 30 April, 2022

Keywords:
Urbanization
Mathematical model
Queueing System
performance measures
waiting line

ABSTRACT

The increase in land travel demand over the years which poses a challenge in managing the high influx of people and goods into urban centres and cities such as Abuja, Calabar, Lagos, Kano, and Kaduna, Nigeria has led to queues in security check points. This study determines a mathematical model for analysing queueing system performance measures using the data obtained from Kugbo check-point, Abuja Keffi Highway. The service time data and system settings are used to choose the M/M/1 queueing mathematical model for a single server queueing system. According to the study, an average queue includes 3751 cars and takes 1 hour and 11 minutes to complete. An hourly queueing graph showed that on the busiest days, queue length and waiting time rose. Based on the busiest days (Mondays), an hourly queueing graph revealed that customer queueing length and waiting time is at its highest in the morning hours from 7am to 11am, and then from 4pm to 7pm. For effective and economical system management, this length of time spent on queue should be taken into account, also artificial intelligent simulation models like Netsim, SimEvent in MATLAB models are recommended for comparative studies.

1.0 Introduction

With the 5% annual rate of urbanization, the united nation projected that half of the world population would live in urban areas at the end of 2020 (United Nation, 2016). It is predicted that by the year 2050, approximately 64% of the developing world and 86% of the developed world would be urbanized. That is equivalent to approximately 3 billion urbanites by 2050, much of which will occur in Africa and Asia (United Nation Population Fund 2012). Notably, it is also found time past by United nation that nearly all global population growth from 2017 to 2030 will be absorbed by cities, about 1.1 billion new urbanites over 13 years. Urbanization creates enormous social, economic and environmental changes such as traffic queue, congestion and delay especially in intracity transportation services. This traffic congestion in traffic service delivering points creates queue, hence queue and time lost ought to be studied. The term "queue" can be defined as a waiting line especially of persons or vehicles (customers) in a file or line waiting for services where one or more customers are called upon at a time to be served based on accessible service units known as servers. Queueing theory has its origins in a research that established models to describe the Copenhagen telephone exchange (Chen, 2021). The ideas have since seen applications in telecommunication, traffic engineering and computing and particularly in industrial engineering, design of factories, shops, offices and hospitals, as well as in project management. It is employed in transportation facility to ensure quality control. With this knowledge, it is possible to rationalize the decision making process, with respect to waiting lines. From a qualitative one, thereby improving the chances of deciding correctly (Agner, 1909). Service pattern – series called' "service discipline" where identical servers are accessed through two or more channels rendering the same services; and queueing rule refers to as queue discipline which include; First-In-First-Out (FIFO) meaning, who comes in first leaves earlier, Last-In-First-Out (LIFO) meaning, who comes in later leaves earlier. Random Order of Service (ROS) which means random selection of customers. Shortest Processing Time First (SPTF) meaning, customers with shortest service processing time will receive service first and Longest Processing Time First (LPTF) meaning, ROS opposite. A priority queue is that in which arrivals are classified into groups based on criterion (Adeke, 2015; Borgs et al., 2012; Penttinen, 2008; Daigle, 2005; Sethi et al., 2020, Kleinrock, 1975). Queueing process is basically presented as shown in Figure 1, where arrivals enter the system, they join a queue, wait for services and then proceed into the service facility to be served after which they depart out of the system.

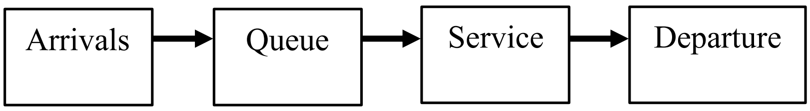

Figure 1: Schematic of Queueing Process *(Adeke, 2015)*

Following the increasing demand for land travel over the years, its poses a challenge in managing high inflow of vehicles into cities, which result into queue in the security check-in system which leads to prolong waiting time (Biliyamin and Abosede, 2012). The check-in process requires vehicles to join queues for security check using scanners, which could lead to congestions and tension on the road if not properly managed. Queueing studies is aimed at using queueing parameters such as; inter-arrival, pattern of service, service rate, busy and idle times of server and number of service units, to examine performance characteristics of system service, and other parameters of interest called "performance characteristics", this is used for determining the efficiency and capacity management strategy for the system (Zukerman, 2018). The performance characteristics is pivotal for every queueing studies. The performance of queueing system is influenced by system configuration and service protocol which is significant to system management in decision making (Adeke, 2015, Varma and Maguluri, 2022).

Following the 5% annual growth in travel demand, it can be related that the future of every checkpoint, bus stops, and other related bottlenecks are at risk. Queue affects customers' value of time (VOT), and it forces them to renounce more productive and rewarding way of using time and to renege promises.

This study attempts to determine queueing system performance characteristics (waiting time, queue length) using mathematical models. To justify the goal of this research, the daily growth in land traffic response and terrible increase in security checkpoints make it necessary to create a model to anticipate the future of vehicle performance in various checkpoints.

2. Material and Methods

2.1 Probability and distribution function

Consider the case where the inter-arrival and service times are independent and exponentially distributed (memoryless). Therefore, the arrival process follows a Poisson process with parameter λ and service times are assumed to be IID and exponentially distributed with parameter μ, and are independent of the arrival process. Queueing entities are random variables; When we say that random variable X takes value x, this means that x represents a certain outcome of an experiment which is an event, so {X = x} is an event. Therefore, we may assign probabilities to all possible values of the random variable. Therefore the function denoted $P_X(x) = P(X = x)$ is called probability function. The cumulative distribution function of random variable X is defined for all $X \in R$ (R being the set of all real numbers), is defined as;

$$F_X(x) = P(X \leq x)$$

Accordingly, the complementary distribution function $\bar{F}_X(x)$ is defined by;

$$\bar{F}_X(x) = P(X > x)$$

Consequently, for any random variable, for every $X \in R$, $F(x) + \bar{F}(x) = 1$. This relation is termed a reversible process. Then $Y = g(X)$ is also a random variable. In this case, if $P_X(x)$ is the probability function of X then the probability function of Y is;

$$P_Y(y) = \sum_{x;g(x)=y} P_X(x)$$

A Poisson random variable with parameter λ has the following probability function:

$$P(X = i) = e^{-\lambda}\frac{\lambda^i}{i!} \quad i = 0, 1, 2, 3, \ldots.$$

To compute the values of $P(X = i)$, it may be convenient to use the recursion

$$P(X = i + 1) = \frac{\lambda}{i+1} P(X = i)$$
$$P(X = 0) = e^{-\lambda} \quad (1)$$

The importance of the Poisson random variable lies in its property to approximate the binomial random variable in case when n is very large and p is very small so that np is not too large and not too small. For convenient we will consider a sequence of binomial random variables Xn, n = 1, 2, . . . with parameters (n, p) where $\lambda = np$, or $p = \lambda/n$. Then the probability function $\lim_{n\to\infty} P(X_n = k)$ is a Poisson probability function with parameter λ.

To prove this we write:

$$\lim_{n\to\infty} P(X_n = k) = \lim_{n\to\infty} \binom{n}{k} P^k(1-p)^{n-k}$$

Substituting $p = \lambda/n$, we obtain

$$\lim_{n\to\infty} P(X_n = k) = \lim_{n\to\infty} \frac{n!}{(n-k)!\,k!}\left(\frac{\lambda}{n}\right)^k \left(1 - \frac{\lambda}{n}\right)^{n-k}$$

Now notice that $\lim_{n\to\infty}\left(1 - \frac{\lambda}{n}\right)^n = e^{-\lambda}$

$$\lim_{n\to\infty}\left(1 - \frac{\lambda}{n}\right)^{-k} = 1$$

And $\lim_{n\to\infty} \frac{n!}{(n-k)!n^k} = 1$

Therefore, $$\lim_{n\to\infty} P(X_n = k) = \frac{\lambda^k e^{-\lambda}}{k!} \quad (2)$$

$$f(x) = \{X \le x\} = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

The cumulative distribution function (cdf) is given as

$$F(x) = P\{X \le x\} = \begin{cases} 1 - e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

The general relationship between these functions is;

$$f(x) = \frac{dF(x)}{dx} \quad (3)$$

Equation (3) is according to Zukerman (2018).

2.2 Utilization

Utilization is an important measure for queueing systems performance, denoted by ρ. It is the average proportion of time a server is busy. In most scenarios a server pays for its time regardless of whether it is busy or not. Normally, the time that transmission capacity is not used is time during which money is spent but no revenue is earned. It is, therefore, important to design systems that will maintain high utilization (Zukerman, 2018). If you have two identical servers and one is busy 0.4 of the time and the other 0.6. Then the utilization is 0.5. We always have that $0 \le \rho \le 1$. If we consider an M/M/∞ queue (Poisson arrivals, exponentially distributed service times and infinite servers) and the arrival rate is finite, the utilization is zero because the mean number of busy servers is finite and the mean number of idle servers is infinite, therefore queue won't build up. In scenarios where M/M/1 queue is considering the utilization equation would be ρ>0 (Zukerman, 2018).

Therefore Utilization Equation $$\rho = \frac{\lambda}{\mu} \quad (4)$$

This expression is called traffic intensity or Utilization equation. Figure 3 presents a system of queueing with vehicular arrival rate, λ and service rate as μ in a sequence of arrival, in a single server customer flow describing a utilization factor.

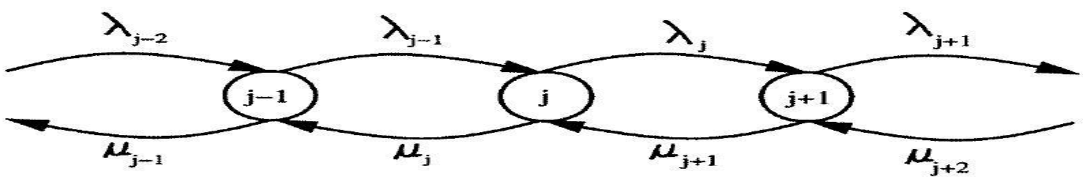

Figure 3: Flow Diagram for M/M/1 Model *(Daniel, 1995).*

2.3 Little's Formula

The fundamental queueing formula $L = XW$ (Little's law) states that

> "The time-average queue length (number in the system) L is equal to the product of the arrival rate X and the customer-average waiting time (time spent in the system) W".

This formula is valid in great generality, and defined by Little (1961).

This is an important queueing mathematical theory result that applies to M/M/1 queue (and to the other systems) is called Little's Formula. It can also be expressed as;

$$E[Q] = \lambda E[D] \quad (5)$$

Where $E[Q]$ and $E[D]$ are the representative of the stationary mean queue-size and the delay mean (system waiting time) from the moment it arrives until the culmination of the service, respectively (Zukerman, 2018). This is graphically displayed in Figure 2.

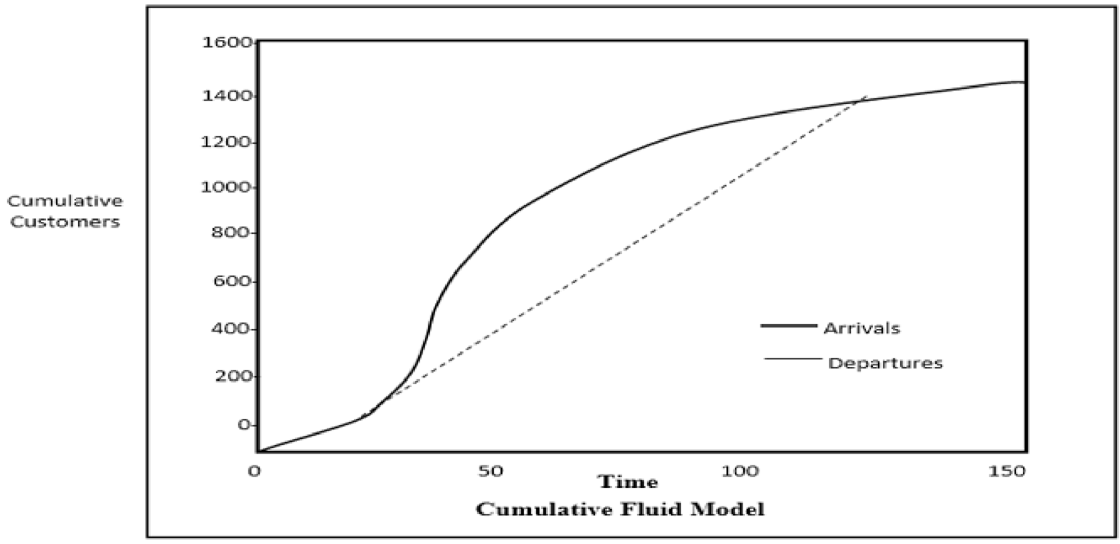

Figure 2: Cumulative Fluid Model (Adeke 2015)

From the graphical proof of Little's Formula for the case of M/M/1. Consider a stable M/M/1 queue that starts at time t=0, for a null queue. Let $A(t)$ be the number of arrivals up to time t, and let $D(t)$ be the number of departure up to time t. The queue-size (number in the system) at time t is denoted $Q(t)$ and is given by

$$Q(t) = A(t) - D(t), t \geq 0 \qquad (6)$$

Let L be an arbitrarily long period of time. Then the mean queue-size $E[Q]$ is given by

$$E[Q] = \frac{1}{L}\int_0^L Q(t)dt \qquad (7)$$

Also notice that

$$\int_0^L Q(t)dt = \sum_{i=1}^{A(L)} W_i$$

Where W_i represent the time spent in the system by the ith customer. (Since L is arbitrarily large, the may have been large number of events during [0, L] where our stable M/M/1 becomes empty, so $A(L) = D(L)$.) Therefore,

$$\frac{1}{L}\int_0^L Q(t)dt = \frac{1}{L}\sum_{i=1}^{A(L)} W_i$$

And realizing that $\lambda = A(L)/L$

And $E[D] = \frac{1}{A(L)}\sum_{i=1}^{A(L)} W_i$ (8)

$$E[Q] = \frac{1}{L}\int_0^L Q(t)dt = \frac{A(L)}{L}\frac{1}{A(L)}\sum_{i=1}^{A(L)} W_i = \lambda E[D].$$

Considering the number `of customers in the systems, denoted by N_s. Where N_s can only take the values of zero or more,

$$E[Q] = \lambda E[D].$$

$$E[N_s] = \lambda E[D].$$

where $E[D] = 1/\mu$; $E[N_s] = \lambda(1/\mu)$

$$E[N_s] = \frac{\lambda}{\mu}$$

The conventional Notations in queueing theory for k-server queue is represented by Equation (4).

Since queue process is a stochastic process that obeys birth and death process. In M/M/1 we consider a case where the arrival process follows poisons process with parameter λ and service times are assumed to be independently identically distributed with parameter μ and is independent of the arrival (Zukerman, 2018). Assuming M/M/1 queue capacity starts from j, the first entity will pass through state n, for a period of time λ (λ =0, because no queue). Then it moves to state $j+1$. The total time the entity stays in the system is also exponentially distributed and is independent of any arrival = μ.

The second entity enters state j and wait until the first entity is done with service. After the second entity spends the total time of $\lambda + \mu$, it will move to state $j+1$ with probability $\frac{\lambda}{\lambda+\mu}$ and to state $j-1$ with probability $\frac{\mu}{\lambda+\mu}$.

We write that; $\pi_{j+1}(\lambda + \mu) = \pi_j(\mu)$

$$\pi_{j+1}\mu = \pi_j \lambda \qquad \text{for I = 0, 1, 2, 3}$$

From equation 6, $\rho = \frac{\lambda}{\mu}$

$$\pi_{j+1} = \pi_j\, \rho$$

Since the Utilization equation from j to j+1 is ρ, the opposite will be

$$\pi_0 = 1 - \rho$$

$$\pi_1 = \pi_0\, \rho$$

$$\pi_1 = \rho(1 - \rho)$$

Summing the states we obtain; $E[Q] = \sum_{i=0}^{\infty} i\pi_i$.

$$E[Q] = \frac{\rho}{1-\rho}.$$

According to Zukerman (2018), Queue Length, $L = \frac{\rho}{1-\rho}$ (9)

2.4 Data Source

Traffic count data were collected at Kugbo checkpoints. The daily vehicular demand was 45000 units and hourly vehicular demand was 3698 units. Figure 4 below presents the traffic situation in Kugbo checkpoint.

Figure 4: Traffic Situation in Kugbo Checkpoint showing the presence of queue

2.5 Data Collection

Traffic count data were collected using a pneumatic sensor traffic dictator connected to a computer, which reads the fluid pressure from the vehicles. The data collected are the arrival time and the service time.

2.6 Data Processing

The arrival rate was computed using Equation (12) and service rate was determined using Equation (13). Easyfit statistical application was used to determine the frequency distribution that the arrival time and service rate followed.

2.7 Assumptions

The following assumptions were applicable to scenarios considered for the study.

- Arrivals are discrete and assumed to follow poisons distribution and the system assumes exponentially distributed service time.
- The system assumes a steady state with utilization factor or traffic intensity $\rho_t < 1$.
- The scenario operates using FIFO and non-pre-emptive queueing rules
- The queue server is assumed to be one
- There is inspection and rejected interjecting arrivals in the system.

2.6 Basic Analysis

Following Adeke (2018), for the estimation of the parameters, the period of heavy operation was assumed from the day with the busiest traffic shown in Table 3, to be the peak period of this analysis; that is a non-stop inflow and continues service situation spanning from 06:00 am to 06:00 pm.

$$\text{Inter-arrival time,(sec/veh)} = \frac{12*60*60(sec)}{Average\ Arrival\ Demand\ (\text{vehs})} \tag{10}$$

$$\text{Service time, (sec/veh)} = \frac{12*60*60\ (sec)}{Average\ daily\ Departure\ (\text{vehs})} \tag{11}$$

$$\text{Arrival rate, } \lambda \text{ (veh/sec)} = \frac{Average\ Arrival\ Demand\ (\text{vehs})}{12*60*60\ (sec)} \tag{12}$$

$$\text{Service Rate, } \mu \text{ (vehs/sec)} = \frac{Average\ daily\ Departure\ (\text{vehs})}{12*60*60\ (sec)} \tag{13}$$

Summary of the Equations utilized in the analysis is presented in Table 1.

Table 1: Summary of Performance Estimation Formulae

Performance parameters	Formulae
Utilisation factor	$\rho = \frac{\lambda}{\mu}$
Average waiting time in system	$W_q = \frac{1}{\mu(1-\rho)}$
Average queue length	$L_q = W\lambda = \frac{\rho}{1-\rho}$

Source: Zukerman (2018)

4. Results and Discussion

4.1 Vehicular Demands

The daily and hourly vehicular travel demands for the Kugbo check point are presented in Table 2. This shows that the volume flow rate of vehicles at this check point is large.

Table 2: Vehicular Travel Demand for Kugbo, checkpoints

S/N	Checkpoints	Estimated Daily Arrivals (Vehicles/Day)	Estimated Hourly Arrivals (Vehicles/hour)
1	**Kugbo**	44379	3698

4.2 Queue System Performance

The waiting time and queue length of customers determined using Little's formula and the performance indicators for Kugbo checkpoint, are presented in Table 3.

Table 3: Daily Vehicular Performance Measures

Days	TotalArrival (Veh)	Arrival Rate (Vehs/sec)	Service Rate (Veh/sec)	Traffic Intensity (unit)	Average waiting Time(Sec)	Queue Length (Veh)
Monday	42443	0.9825	0.9985	0.9840	3818	3751
Tuesday	39598	0.8950	0.9985	0.89635	84	75
Wednesday	38069	0.8612	0.9985	0.8625	46	39
Thursday	36862	0.8008	0.9985	0.8020	20	16
Friday	39378	0.8555	0.9985	0.8868	62	56
Saturday	34996	0.6965	0.9985	0.6975	7	5
Sunday	34760	0.6169	0.9985	0.6179	4	3

Table 3 shows the service rate for all days, and assumed to be the same for all days, the traffic intensity (utilization factor), waiting time and queue length of various days was estimated. Monday has the highest traffic intensity, waiting time and queue length followed by Tuesday, Friday and Wednesday, Sunday has the least queueing effect. These performance characteristics are dependent on the arrival rate, the higher the arrival rate the higher the performance characteristics. From the traffic utilization factors, this show that Monday has the busiest traffic as well as the longest queue length of 3751 vehicles and waiting for 3818 seconds in other words 64 minutes, a whole lot of time to waste. This signals a problem to motorists plying this route. It is practically impossible to have every hour on Monday very busy. Table 4 displays the evidence of cumulative hourly queue distribution on Monday. The arrival of vehicles at the checkpoint are different depending on the hour of arrival and the service rate is assumed a constant at 3594 hourly service time taken on average, finally the cumulative queue shows the difference between the arrival and service describing the utilization factor where by, the negative sign indicates no queue while otherwise queue.

Table 4: Vehicular Hourly Cumulative Arrival and Departure November 2017

Time(h)	7	8	9	10	11	12	13	14	15	16	17	18
Arrival	1855	12213	18527	22291	24740	28456	31708	34083	36020	37270	39526	38833
Service	3594	7193	10841	14318	18018	21706	25304	28905	32497	36018	39606	39526
Cum. Queue	-1739	5020	7686	7973	6722	6750	6404	5178	3523	1252	-80	-693

The bar differences between the constant and the rising arrival indicate the queued vehicles. At the last column of 18 hour of the day, the service rate rose higher than the arrival rate indicating the reconciliation period, where the queue dissipates. Consequently, the idle period in the system is indicated with a negative sign which denotes under-utilization. It is seen that between 7am to 3pm the queue builds up and collapsed by 4pm, but the peak of it is by 8 to 10am. Therefore, the waiting line becomes infinite around its peak hours. Every hour has her traffic situation at a constant service rate, and this is demonstrated in Figure 5 which shows that the hourly queue condition is dependent on the rate of arrival.

Figure 5: Hourly Arrival and Service Graph

Following the information in Figure 5, the waiting line is over 7830 vehicles which is equivalent to 7970 seconds, 2 h and 13 mins. Therefore, within this time customer are delayed for quite an unbearable time. This can be related to customer's feelings and reaction to services delivered lately which neutralizes potential happiness of the service delivered. This study is relevant to urban settlers in decision making for optimum service delivery.

5. Conclusion

The queueing system performance characteristics were determined using mathematical models and it was found that; an average queue length of 3751 vehicles and, 1h and 11mins waiting time. Also the customers suffer more queue on Mondays. Judging from the busiest day an hourly queue chart proved that the queue length and waiting time of customers is at its peak in morning hours from 7am to 11 am and subsequently from 4pm to 7pm. `looking at this, Commuters could be late for Jobs, Truckers could incur additional charges and business travellers can loss their transaction. For effective and economical system management, this length of time spent on queue should be taken into account. Other artificial intelligent simulation models like Netsim, SimEvent in MATLAB models are recommended for comparative studies.

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